**1.**

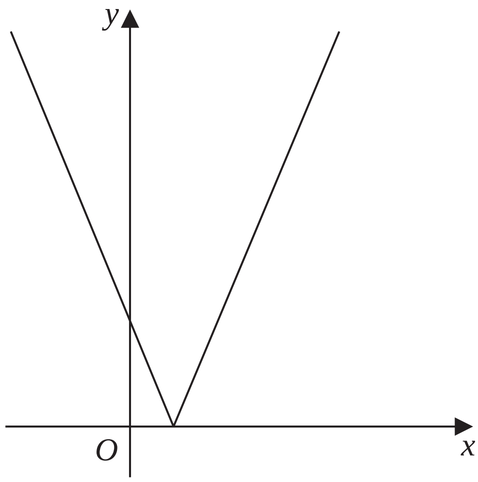


**Figure 4**

Figure 4 shows a sketch of the graph with equation

$$y = |2x - 3k|$$

where $k$ is a positive constant.

(a) Sketch the graph with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = k - |2x - 3k|$$

stating

- the coordinates of the maximum point
- the coordinates of any points where the graph cuts the coordinate axes

**(4)**

(b) Find, in terms of $k$, the set of values of $x$ for which

$$k - |2x - 3k| > x - k$$

giving your answer in set notation.

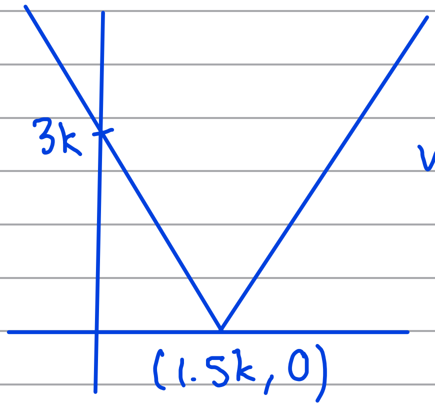
**(4)**

(c) Find, in terms of $k$, the coordinates of the minimum point of the graph with equation

$$y = 3 - 5\mathrm{f}\left(\frac{1}{2}x\right)$$

**(2)**

a)



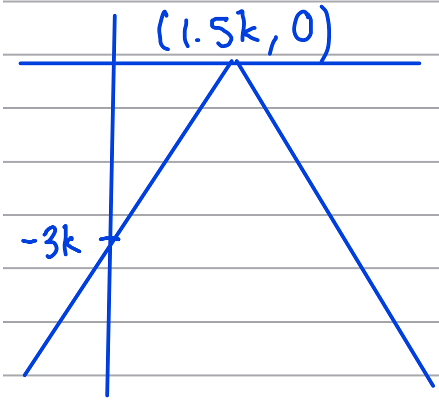

$y = |2x - 3k|$ $\quad k > 0$

vertex: $|2x - 3k| = 0$
$2x - 3k = 0$
$2x = 3k$
$x = 1.5k$

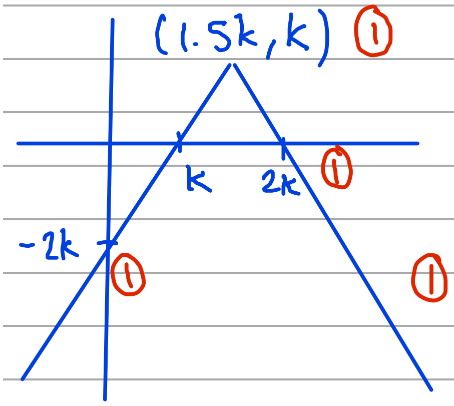
y-intercept:
$y = |0 - 3k|$
$= |-3k|$
$= 3k$

want to sketch $y = k - |2x - 3k|$.

Method: first sketch $y = -|2x - 3k|$, then add $k$.




$y = -|2x - 3k|$ is a reflection of the first graph in the $x$-axis.

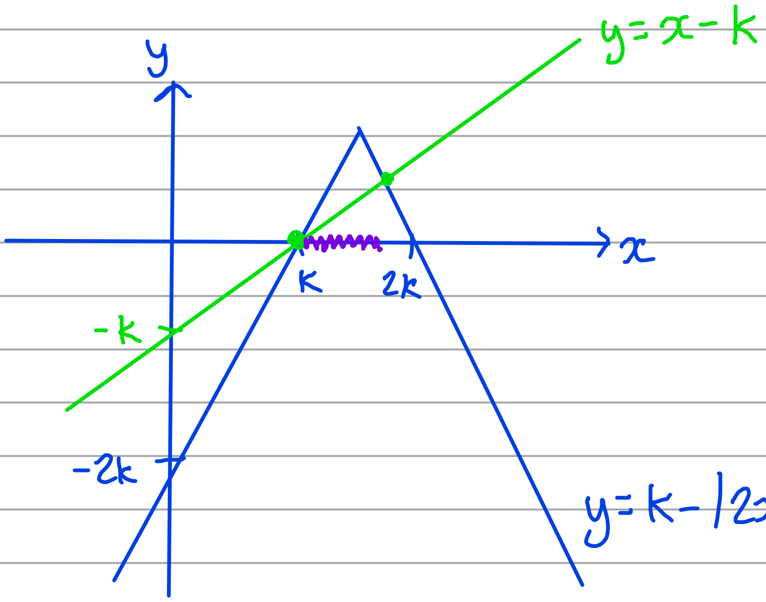



$y = k - |2x - 3k|$ is a translation of $\begin{pmatrix} 0 \\ k \end{pmatrix}$ of the previous graph.

find roots: $k - |2x - 3k| = 0$
$|2x - 3k| = k$

$2x - 3k = k \Rightarrow x = 2k$ $\qquad$ $2x - 3k = -k \Rightarrow x = k$

b) $k - |2x - 3k| > x - k$




mmm = want this region

first critical value is $x = k$ (1)

$\therefore \quad x > k$

Find second critical value:

$k - |2x - 3k| = x - k$ (1)

$|2x - 3k| = -x + 2k$

$2x - 3k = -x + 2k$
$3x = 5k$
$x = \frac{5k}{3}$ (1)

$2x - 3k = x - 2k$
$x = k$
found already

$\therefore \; x < \frac{5k}{3}$

set notation: $\left\{ x : x < \frac{5k}{3} \right\} \cap \left\{ x : x > k \right\}$ (1)

$y = 3 - 5f\left(\frac{1}{2}x\right)$

$f(x)$ has maximum point
$(1.5k, k)$

$f\left(\frac{1}{2}x\right)$ has maximum point
$(3k, k)$

$-5f\left(\frac{1}{2}x\right)$ has minimum point
$(3k, -5k)$

$3 - 5f\left(\frac{1}{2}x\right)$ has minimum point

$(3k, 3-5k)$

(1) (1)

**2.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

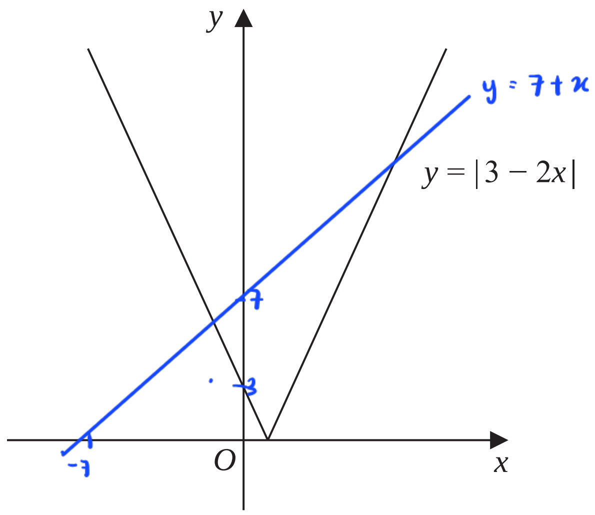



**Figure 1**

Figure 1 shows a sketch of the graph with equation $y = |3 - 2x|$

Solve

$$|3 - 2x| = 7 + x$$

**(4)**

$(3 - 2x)^2 = (7 + x)^2$

$9 - 12x + 4x^2 = 49 + 14x + x^2$ (1)

$3x^2 - 26x - 40 = 0$ (1)

$(3x + 4)(x - 10) = 0$

either $3x + 4 = 0$ , $x - 10 = 0$ (1)

$x = \frac{-4}{3}$ , $x = 10$ (1)

**3.**

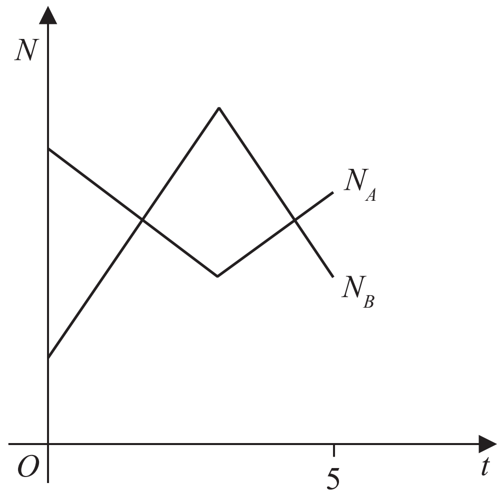



**Figure 2**

The number of subscribers to two different music streaming companies is being monitored.

The number of subscribers, $N_A$, in thousands, to **company A** is modelled by the equation

$$N_A = |t - 3| + 4 \qquad t \geqslant 0$$

where $t$ is the time in years since monitoring began.

The number of subscribers, $N_B$, in thousands, to **company B** is modelled by the equation

$$N_B = 8 - |2t - 6| \qquad t \geqslant 0$$

where $t$ is the time in years since monitoring began.

Figure 2 shows a sketch of the graph of $N_A$ and the graph of $N_B$ over a 5-year period.

**Use the equations of the models to answer parts (a), (b), (c) and (d).**

(a) Find the initial difference between the number of subscribers to **company A** and the number of subscribers to **company B**.

**(2)**

When $t = T$ **company A** reduced its subscription prices and the number of subscribers increased.

(b) Suggest a value for $T$, giving a reason for your answer.

**(2)**

(c) Find the range of values of $t$ for which $N_A > N_B$ giving your answer in set notation.

**(5)**

(d) State a limitation of the model used for **company B**.

**(1)**

a) initial ⇒ $t=0$

$N_A = |0-3| + 4 = 7$ $\quad$ $N_B = 8 - |0-6| = 2$

$N_A - N_B = 5$ (1) $\therefore$ difference is 5000 subscribers (1)

b) find vertex of graph:

$N_A = |t-3| + 4$

vertex has $|T-3| = 0 \quad \therefore T = 3$ (1)

This was the point where the number of subscribers for A started increasing. (1)

c) find critical values

(1)

$|t-3| + 4 = 8 - |2t-6|$

$|2t-6| = |2(t-3)|$
$= |2||(t-3)| = 2|t-3|$

$|t+3| + 4 = 8 - 2|t-3|$

$3|t-3| = 4$ (1)

$|t-3| = \frac{4}{3}$

$t - 3 = \frac{4}{3}$ $\qquad$ $t-3 = -\frac{4}{3}$

$t = \frac{13}{3}$ (1) $\qquad$ $t = \frac{5}{3}$

choose the outside region: $t < \frac{5}{3}$ or $t > \frac{13}{3}$ (1)

$\left\{ t \in \mathbb{R}^+ : t < \frac{5}{3} \right\} \cup \left\{ t \in \mathbb{R}^+ : t > \frac{13}{3} \right\}$ (1)

$\mathbb{R}^+$ = positive real numbers

d) eventually, the number of subscribers for B will become negative. 1